CO2 – A CHAMP Magnetic Field Model

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Summary. We describe here a model of the magnetic field created specifically to be used with data from the CHAMP satellite, valid for the initial period of the CHAMP mission (July 2000 - December 2001). The model contains initial field, linear secular variation and external field contributions. Although CHAMP data provide the most important input to the modelling, we also use data from other satellites (\textsc{Orsted} and \textsc{Orsted-2}/\textsc{SAC-C}) and ground-based observatories, to increase the robustness of the model. The model will be improved upon in the future (in particular as we better understand the local time dependencies of the external and induced magnetic fields) but for now is the recommended standard model for use with applications of CHAMP data.

Key words: magnetic field modelling, modelling methodology

1 Model parameterization

We describe here briefly the modelling methodology (for more detail, see \cite{[4]}). We represent the magnetic field as the gradient of a scalar potential \(\mathbf{B} = -\nabla \Phi\) where

\[
\Phi = a\left\{ \sum_{n=1}^{N_{MF}} \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) \left(\frac{a}{r}\right)^{n+1} P_n^m (\cos \theta) \\
+ \sum_{n=1}^{N_{SV}} \sum_{m=0}^{n} (\dot{g}_n^m \cos m\phi + \dot{h}_n^m \sin m\phi) (t - t_0) \left(\frac{a}{r}\right)^{n+1} P_n^m (\cos \theta) \\
+ \sum_{n=1}^{2} \sum_{m=0}^{n} (q_n^m \cos m\phi + s_n^m \sin m\phi) \left(\frac{r}{a}\right)^n P_n^m (\cos \theta) \\
+ D_{st} \cdot \left[ \left(\frac{r}{a}\right)^2 + Q_1 \left(\frac{a}{r}\right)^2 \right] \times \left[ \tilde{q}_1^0 P_1^0 (\cos \theta) + (\tilde{q}_1^1 \cos \phi + \tilde{s}_1^1 \sin \phi) P_1^1 (\cos \theta) \right] \right\}.
\]

\((r, \theta, \phi)\) are the standard Earth-centred spherical coordinates (radius, colatitude, longitude), with a reference Earth radius of \(a = 6371.2\) km. \(P_n^m\) are the associated Legendre polynomials of degree \(n\) and order \(m\). \(t\) is time in years,
with the model epoch defined $t_0 = 2001.0$. \{$g_n^m, h_n^m\}$ are the internal Gauss coefficients (calculated to degree $N_{MF} = 29$), \{$g_n^m, h_n^m\}$ the coefficients of main field secular variation (calculated to degree $N_{SV} = 13$), and \{$q_n^m, s_n^m\}$ (calculated to degree 2) the large scale external field coefficients. The coefficients $\tilde{c}_1, \tilde{q}_1^0$ and $\tilde{s}_1^0$ account for the variability of contributions from the magnetospheric ring current (parameterised for ease of use by the $Dst$ index) plus their internal, induced counterpart (considered by the factor $Q_1 = 0.27$). The $n = 1, 2, m = 0$, terms incorporate an annual and semi-annual variation:

$$q_n^0(\tau) = q_{n,0}^0 + (q_{n,1c}^0 \cos \tau + q_{n,1s}^0 \sin \tau) + (q_{n,2c}^0 \cos 2\tau + q_{n,2s}^0 \sin 2\tau)$$

(and similar for $g_n^0$) for $n = 1$ and $2$, where $\tau = 2\pi(t - t_0)$. In total, the model has 1121 free parameters.

It is most common to determine the model coefficients by a simple least-squares fit to the data. This approach assumes implicitly that the errors on the data are Gaussian distributed, but in practice the distribution of misfit to a model has much longer tails. To account for this, we use Iteratively Reweighted Least Squares with Huber weights ($c = 1.5$) [4]. Examination of data residuals justifies this choice a posteriori.

## 2 Data

We use CHAMP and Ørsted scalar and vector data between August 2000 and December 2001, and Ørsted-2 scalar data between January 2001 and December 2001. CHAMP data were selected preferentially to ensure that the model is most appropriate for CHAMP applications. We use the $Kp$ index to restrict the data to quiet times, specifically requiring $Kp \leq 1^+$ for the time of observation and $Kp \leq 2^o$ for the previous three hour interval. CHAMP data are further restricted to extended quiet-time periods ($Kp \leq 1^+$ for at least one day) to allow an independent calibration of the transformation of the vector measurements into Earth-centred coordinates. We use the $Dst$ index to define periods when the large-scale external field is weak and stable, requiring $Dst$ within $\pm 10$ nT and $|d(Dst)/dt| < 3$ nT/hr. The effect of polar cap ionospheric currents is minimised by excluding data in the polar caps for which the dawn-dusk component of the interplanetary magnetic field was $|B_y| > 3$ nT. Only night-side data (between 18:00 and 06:00 Local Time) were used, to reduce contributions from ionospheric currents at middle and low latitudes. Vector data have been taken for dipole latitudes equatorward of $\pm 50^\circ$, scalar data were used for regions poleward of $\pm 50^\circ$ or if attitude data were not available. Sampling rate was 60 seconds; weights $w \propto \sin \theta$ are applied, to simulate an equal-area distribution. Ørsted vector data show anisotropic errors due to attitude uncertainty from calculating orientation from only one star camera [5], as do CHAMP data in a dawn-dusk orbit when one of its two cameras is sun-blinded: this is explicitly modelled [2, 1] in the inversion.
In addition to the satellite data, we also use observatory data (linear estimates of secular variation, obtained from quiet periods of the years 1998-2000) to constrain further the secular variation coefficients \( \{g_n^m, h_n^m\} \).

### 3 Model

The model obtained from this analysis is named C02 (standing for CHAMP and the two Ørsted satellites from which the bulk of the data are taken). It is available from the authors, and also from the CHAMP data centre web site isdc.gfz-potsdam.de/champ. The model shows broad agreement with previous models; in particular, the internal field component with \( n > 14 \) shows very good agreement with a dedicated model of the long-wavelength lithospheric field [3] derived from all available CHAMP scalar data.

The statistics of the fit of the model to the data are given in Table 1. The fit is within expectations in a root mean squares sense. The residuals are calculated in a camera boresight coordinate system to allow for anisotropic attitude error in Ørsted and some CHAMP data [1]. The high percentage resolution of model parameters by CHAMP data also arises because of CHAMP’s lower orbit compared with Ørsted and SAC-C, leading to a much greater sensitivity to large \( n \) coefficients. Of the 1121 model parameters, 500 come from the lithospheric field, degrees \( n = 20-29 \).

#### Table 1. Statistics of model fit to data.

Resolution totals (percentage of parameters resolved by each data source) combined for all data types. \( F_{\text{polar}} \) are from above 50° dipole latitude. Vector data considered in \((B, \perp, 3)\) coordinate system, where \( B \) is the local field direction, \( \perp \) the direction perpendicular to both \( B \) and the star camera boresight axis, and 3 the third direction perpendicular to \( B \) and \( \perp \). Although most CHAMP data are unaffected by attitude uncertainty, for ease of presentation a CHAMP “virtual boresight” is defined in the radial direction.

<table>
<thead>
<tr>
<th>resolution</th>
<th>component</th>
<th>( N )</th>
<th>mean (nT)</th>
<th>rms (nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAMP</td>
<td>( F_{\text{polar}} )</td>
<td>21646</td>
<td>-0.68</td>
<td>6.99</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{nonpolar}} + B_B )</td>
<td>62765</td>
<td>-0.38</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>( B_\perp )</td>
<td>31595</td>
<td>0.01</td>
<td>4.01</td>
</tr>
<tr>
<td></td>
<td>( B_3 )</td>
<td>31595</td>
<td>0.02</td>
<td>4.44</td>
</tr>
<tr>
<td>Ørsted</td>
<td>( F_{\text{polar}} )</td>
<td>22561</td>
<td>-1.57</td>
<td>6.83</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{nonpolar}} + B_B )</td>
<td>35223</td>
<td>0.03</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>( B_\perp )</td>
<td>18913</td>
<td>-0.64</td>
<td>6.66</td>
</tr>
<tr>
<td></td>
<td>( B_3 )</td>
<td>18913</td>
<td>0.11</td>
<td>3.51</td>
</tr>
<tr>
<td>Ørsted-2</td>
<td>( F_{\text{polar}} )</td>
<td>13582</td>
<td>-2.47</td>
<td>5.72</td>
</tr>
<tr>
<td></td>
<td>( F_{\text{nonpolar}} )</td>
<td>19655</td>
<td>0.56</td>
<td>3.13</td>
</tr>
<tr>
<td>Observatories</td>
<td>( \dot{B}_{r, \text{obs}} )</td>
<td>119</td>
<td>-0.67</td>
<td>7.09</td>
</tr>
<tr>
<td></td>
<td>( \dot{B}_{\theta, \text{obs}} )</td>
<td>119</td>
<td>2.85</td>
<td>6.68</td>
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<tr>
<td></td>
<td>( \dot{B}_{\phi, \text{obs}} )</td>
<td>119</td>
<td>-0.34</td>
<td>5.40</td>
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</tbody>
</table>
Whilst the model residuals are within specification, more careful study of their detailed structure reveals problems. In Figure 1 we show the low-latitude scalar ($F$) residuals as a function of time through the mission. We see that the time parameterisation of the model is clearly insufficient, as there is considerable time-dependent signal remaining in the residuals. Further, these are strongly correlated between the different satellites, and so is unlikely to result from differences in local time of the satellites orbits, but instead from inadequate representation of the axisymmetric component of the ring current.

4 Comparison with a particular data set

These problems with unmodelled external field have important implications for the application of the CO2 model and for future modelling of CHAMP data. To illustrate this, we examine one part of the CHAMP data set in detail, focusing on days 615-620, a quiet period in September 2001. In Figure 2 we show residuals of data to various field models. Top left (a) are the residuals to the CO2 model. A clear signal can be seen in the X and Z residuals. Top right (b) we plot the same residuals, but calculated in a local coordinate system.
defined by the unit vector $\mathbf{E}$ in the expected direction of the external field and its corresponding induced component. The residual is largely confined to the $\mathbf{E}$ direction. Bottom left (c) we use the internal coefficients of the CO2 model, but with freely determined external field. The fit is much better, although there is still a signal in the X residual. This signal is also present for a specially derived degree 13 field model calculated with just this data set (bottom right) (d), although more weakly. It may arise from errors in the orientation determination which are systematic between orbits.

5 Application

5.1 For users:

We recommend the CO2 field model for use with CHAMP data. The main field model and its secular variation estimate appear robust. However, the model of the external field is inadequate. To detrend data for a particular time, the external field coefficients must additionally be solved for. A least-squares fit to minimise the sum of squares of residuals should be adequate.
5.2 For modellers:

The dominant error in the CHAMP data, and also in components of the Ørsted data unaffected by attitude error, arises from unmodelled external field. This error is highly correlated within a particular orbit, and also often between orbits. (This problem is much worse without a simple Dst correction.) We suggest three possible approaches to overcome this problem:

1. A better parameterisation of the ring current, in particular its local time dependence (for work towards this, see [6]).
2. Pass by pass removal of external field (detrending of model residuals), already used successfully in a study of CHAMP scalar data [3]. This approach assumes that the field is stationary in time over an orbit, and also risks removing north-south trending signal.
3. Statistical. As shown above, the external field produces an anisotropic error in the vector field. This can be treated in the same way as attitude error in vector data [2]. The 3x3 data error covariance matrix for a vector triple \((X, Y, Z)\) would be

\[ C_e = \sigma^2 I + \mu^2 EE^T, \]

where \(\sigma\) is an estimate of isotropic error, \(\mu\) the rms amplitude of unmodelled external field, and \(EE^T\) is a dyadic of the unit vector in the direction of the local external field.

How helpful will this be? We estimate here \(\sigma = 2.5\, \text{nT}\) and \(\mu = 6\, \text{nT}\) (same order as Ørsted attitude error uncertainty). However, the high degree of correlation from the large-scale external field suggests an even stronger downweighting (larger \(\mu\)) might be appropriate.

For conventional modelling, combining methods 1 and 3 seems promising.

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References